# MUNICH CENTER FOR MATHEMATICAL PHILOSOPHY
10.11.2022

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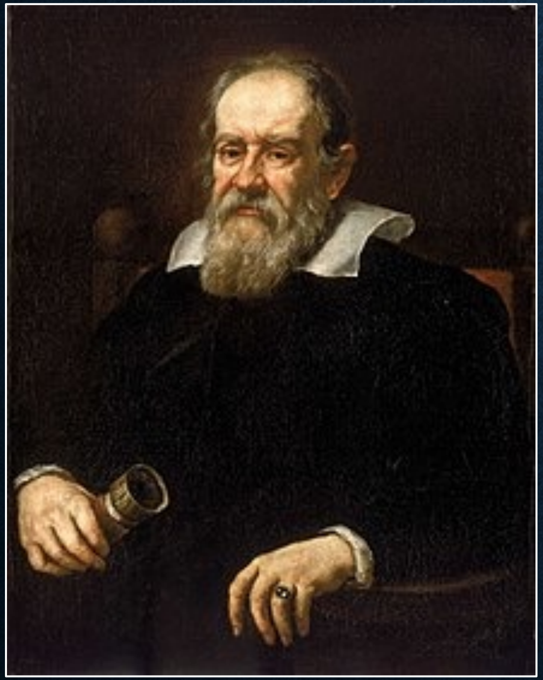
## The development of the complex concept of inertia in the 17th century: Galileo, Descartes, Huygens and Newton




# INTRODUCTION

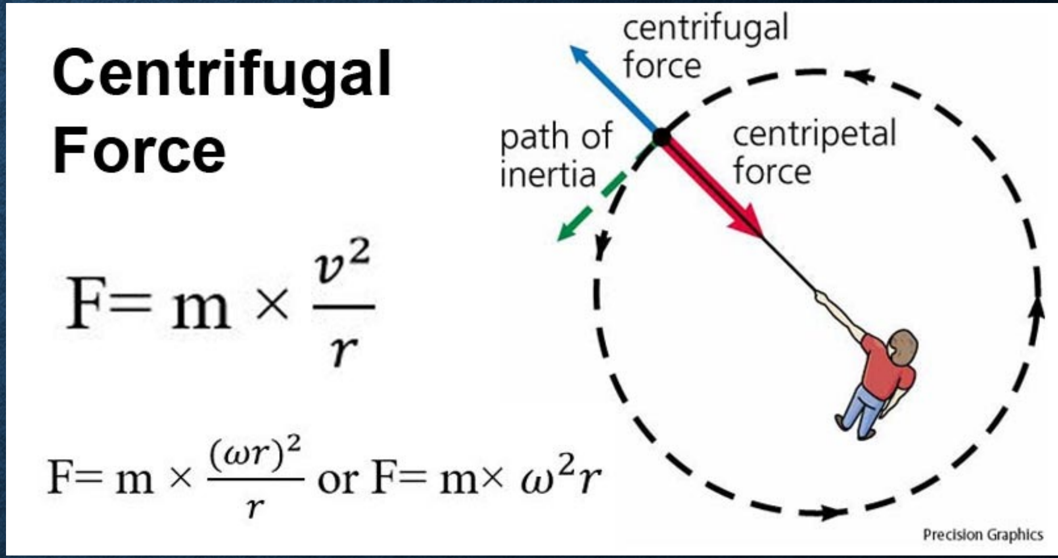
- A motion is defined «inertial» if it continues unmodified without any external action.
- In classical physics the only inertial kinetic states are rest and uniform rectilinear motion (URM). A difficult physical problem consists in determining in respect to what a kinetic state can be regarded as «rest» or URM.
- Aristotle, impetus theory, 17th century scientists, end of the 19th-beginning of the 20th century, general relativity. I will focus on the 17th century scientists.

# INTRODUCTION

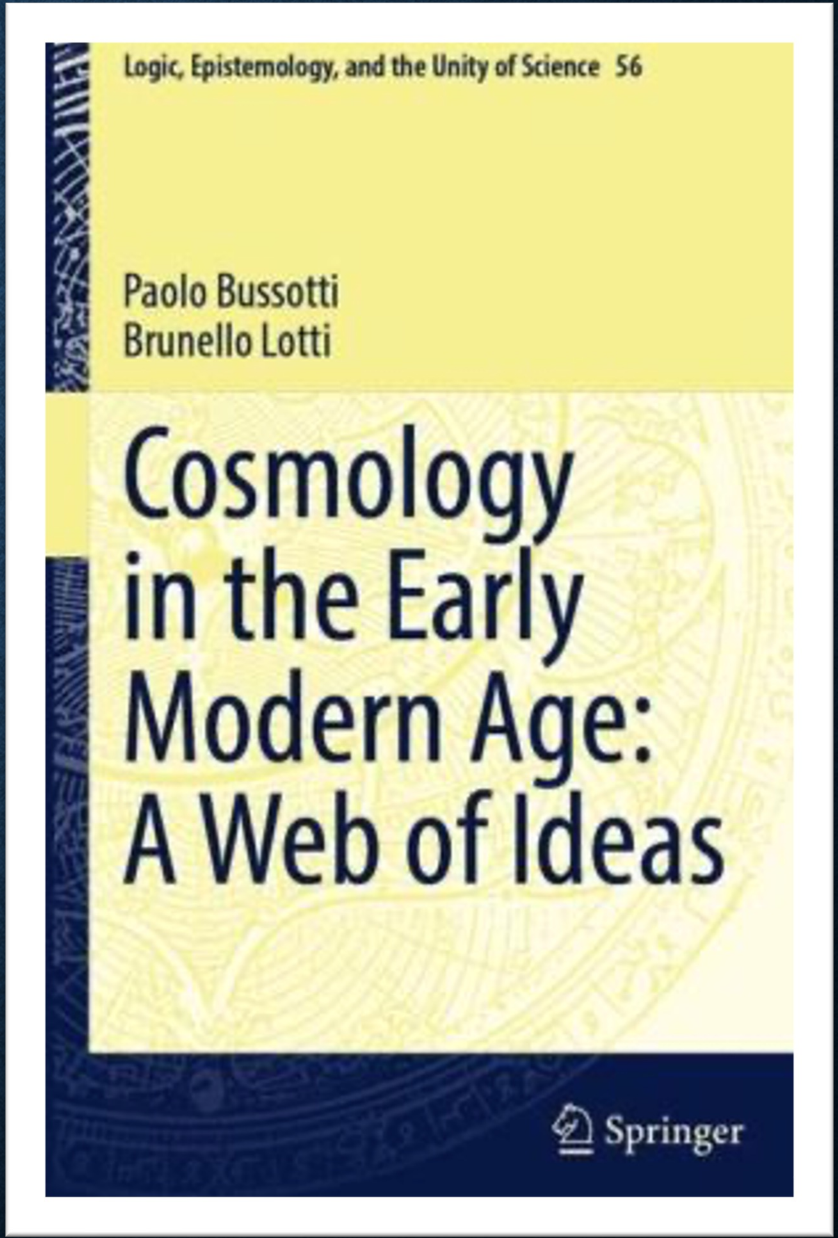
- A force is defined inertial if it is not a real force, but it depends on the inertial state of a body.
- Example 1: you are in a train which is moving uniformly and rectilinearly in respect to the Earth. You feel no force. Suppose that the train accelerates. You are pushed towards your seat. This force is apparent. No force pushes you. The true force is the one which accelerates the train. Instantaneously you tend to maintain your velocity (e.g. 60 km/h) the train accelerates (e.g. from 60 to 80 Km/h) and from your point of view it is as if the train moves towards your with a velocity of 20 Km/h, so that you feel to be pushed towards the seat.
- This is the situation as described in an inertial reference frame. For example, from an observer located out of the train. It is also possible to analyse the situation from your point of view, that is from the point of view of a not inertial observer. In this case the pushing force is real.
- Physics is, in general, described more conveniently from an inertial reference frame.

# INTRODUCTION

- The ball is rotating. In an inertial reference frame, there is the centripetal force due to the guy and the instantaneous inertial tendency. The ball would escape along its instantaneous tangent, but it cannot because of the centripetal force. The sum of these two vectors produces the uniform circular motion.
- You can pose a reference frame comoving with the ball. In this case there is no inertia: The ball experiences a force which acts on it and tends to make it escape radially, but this force is exactly compensated by that due to the man.
- This force is apparent, it is due to the inertia of the ball, no one pushes the ball outside radially.
- Problem: to what can we refer inertia?

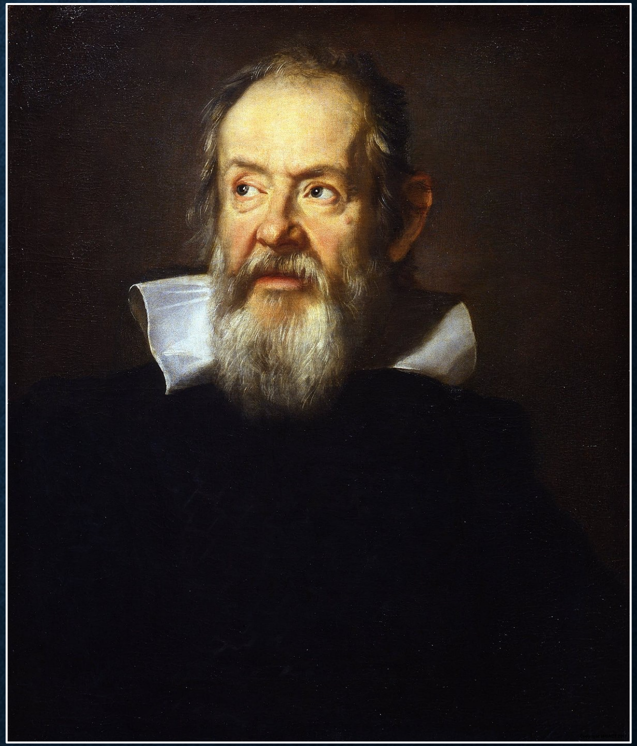


- Next publication, within the end of 2022.



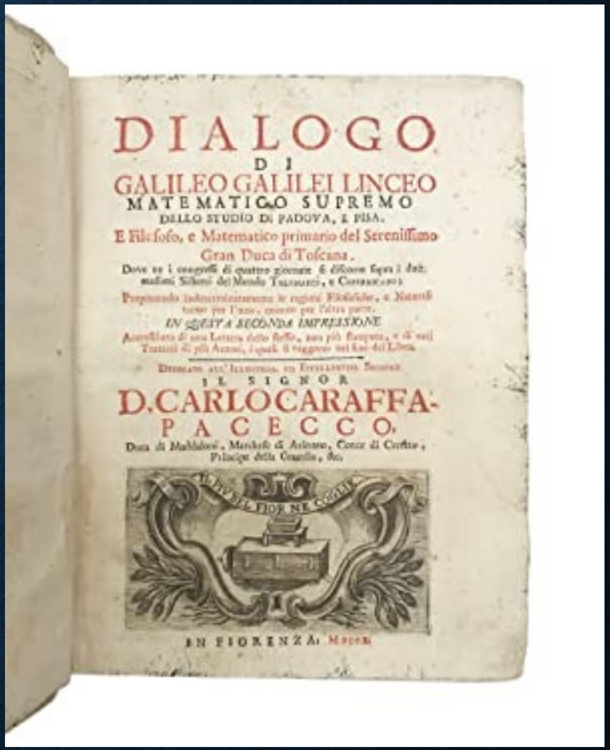
# GALILEO

- Some considerations of Kepler on inertia are interesting, but the scientist who first developed a series of long reflections on this subject was Galileo. I will divide this section into three parts:
- 1) what Galileo wrote;
- 2) a hint to the most important interpretations of Galileo's concept of inertia;
- 3) Lotti's, Pisano's and my interpretation.



# GALILEO

- *Dialogo*: the Platonic myth. Almost at the beginning of the first Day of the *Dialogo*, Salviati claims that the order of the world is perfect: all the positions and movements of celestial bodies are as perfect as possible. Since rectilinear motion is infinite in its nature – because an object moves continuously away from the place where it is – it cannot be the natural motion of celestial bodies:
- […] if all integral bodies in the world are by nature movable, it is impossible that their motions should be straight, or anything else but circular; and the reason is very plain and obvious. For whatever moves straight changes place and, continuing to move, goes ever farther from its starting point and from every place through which it successively passes. If that were the motion which naturally suited it, then at the beginning it was not in its proper place. So then the parts of the world were not disposed in perfect order. But we are assuming them to be perfectly in order; and in that case, it is impossible that it should be their nature to change place, and consequently to move in a straight line. Besides, straight motion being by nature infinite (because a straight line is infinite and indeterminate), it is impossible that anything should have by nature the principle of moving in a straight line; or, in other words, toward a place where it is impossible to arrive, there being no finite end.



# GALILEO

- Therefore, when the cosmos reaches its disposition, no rectilinear motion can take place in the celestial bodies. Such a motion might be used by God in the primordial chaos to transfer the celestial bodies in their appropriate place. But, afterwards, the rectilinear motion ends, and the movement of the celestial bodies is circular uniform. Rectilinear motion is not the proper condition of bodies when they are set in their natural order. It has, instead, a modifying character: by means of it a transition is possible both from order to  disorder and from disorder to order. God used the rectilinear motion only initially, to carry the bodies towards their final natural places and to impart them their speeds. After this initial phase, He converted the rectilinear motion into a uniform circular one, which is maintained without further interventions.

# GALILEO

- Free movement on a portion of the Earth's surface so extended that cannot be approximated by a plane (see the zone in yellow).
- Let us suppose that an impulse be given to a ship on an ocean or a see. How should the motion of the ship continue if the surface of the water were perfectly smooth and no continent or island were interposed? Galileo's answer is that such motion would be uniform circular around a great circle of the Earth; it would be eternal and the crew of the ship would feel no tendency to recede along the tangent. The motion would be horizontal and inertial.
- In *Istoria e dimostrazioni intorno alle macchie solari* (1613 Galileo wrote:

# GALILEO

- "Finally, they [the heavy bodies] are indifferent to some movements—as are the same heavy bodies to horizontal motion—to which they have neither inclination, because it is not toward the center of the Earth, nor aversion, because it is not away from the same center. And therefore, with all the external impediments removed, a heavy body on the spherical surface concentric to the Earth will be indifferent to rest and to movement toward any part of the horizon, and it will remain in the state in which it has been put; that is, if it has been put in a state of rest it will remain in it, and if it has been put in motion, toward the west, for example, it will maintain the same state. And thus a ship, for example, having received one single time some impetus, would move continuously through a quiet sea around our globe without ever stopping, and if one were to bring it gently to rest it would perpetually remain at rest, provided that in the first case all the extrinsic impediments could be removed, and in the second one that no external mobile cause came upon it".

# GALILEO

- Local circular uniform movement. So far we have considered Galileo’s conception of circular motion in a cosmological context and in the case of a uniform circular motion that takes place in a vast portion of the Earth’s surface. Both motions are inertial, according to Galileo. The situation is completely different with regard to the local motion. In this case Galileo recognised that a continuous action is necessary for the circular motion to be maintained. Otherwise, the tendency to recede along the tangent would carry the body along a rectilinear path with a uniform motion. Galilei clearly stated in the second Day of the *Dialogo*:

# GALILEO

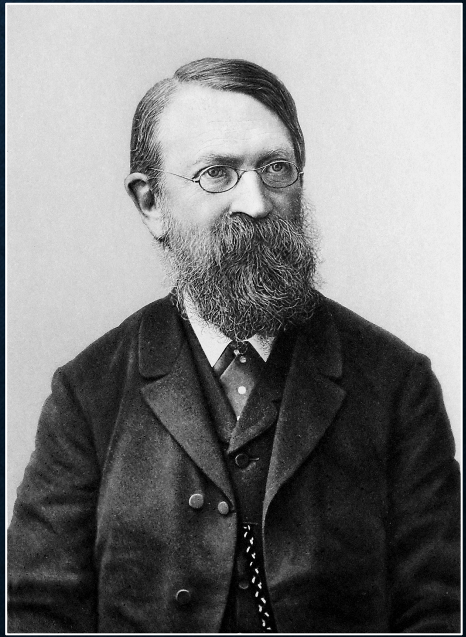
- "Up to this point you knew all by yourself that the circular motion of the projector impresses an impetus upon the projectile to move, when they separate, along the straight line tangent to the circle of motion at the point of separation, and that continuing with this motion, it travels ever farther from the thrower. And you have said that the projectile would continue to move along that line if it were not inclined downward by its own weight from which fact the line of motion derives its curvature".
- This is a perfect description of what happens. The inertia principle is not explicitly formulated, but it is applied correctly, which is even more important. Circular motion is not considered inertial, while uniform rectilinear motion is. If there were no impediments, motion along the tangent would continue ad infinitum. These are the basic properties of inertial motions. Galileo examined also other features of rectilinear uniform motion.

# GALILEO

- Rectilinear inertia: At the beginning of the fourth Day of the *Discorsi e dimostrazioni*, we read:
- Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits.

# GALILEO

- Literature. Galileo’s concept of inertia in secondary literature. Until the studies of Alexandre Koyré (1892-1964) it was generally accepted that Galileo was the discoverer of the inertia principle. In his *Science of mechanics* (first edition 1883), Ernst Mach (1838-1916) paid close attention to the theme of inertia in Galileo. He pointed out how Galileo developed his idea of rectilinear inertia, stressing that, though Galileo had the merit of discovering the inertia principle, he did not use it extensively in his physics.
- Koyré advanced a completely different interpretation: Galileo did not believe in rectilinear inertia. He conceived only inertia of uniform circular motion. Koyré’s reading is linked to his general theses on the scientific revolution which he illustrated in his celebrated work *From the closed world to the infinite universe* (Koyré 1957):



# GALILEO

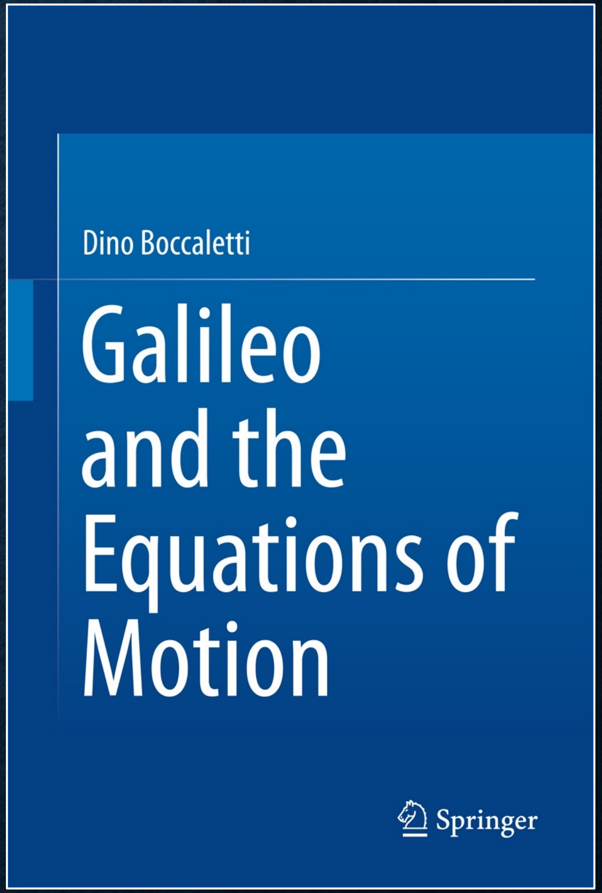
- 1) The scientific revolution was possible because the idea of an ordered and finite cosmos was replaced by that of the infinite universe;
- 2) Mathematics was systematically applied to physics;
- 3) Physicists believed that the laws they were discovering were inherent to the universe;
- 4) Technological and economical factors had scarce influence on the development of theoretical science;
- 5) The experimental aspect was far less important than normally thought of in the birth of modern science.

# GALILEO

- Points 2)-5) represent the Platonic approach of the scientists who lived in the 16th-17th centuries.
- As to Galileo's views on inertia, Koyré thought that he believed only in circular inertia. Galileo was impressed by the natural persistency of circular motion, confirmed by the daily observation of the planets which rotate around the Sun in agreement with his Copernican conviction (Koyré 1939, *Galilean Studies*). According to Koyré, the Platonic myth is the most significant text in Galileo's works that allows us to understand his concept of inertia.
- Koyré also claims that, while referring to the motion along a horizontal plane on the Earth's surface, Galileo was not thinking of a geometrical plane, but of a spherical surface. Inertial motion on a vast part of the Earth's surface is, therefore, circular, not rectilinear. Galileo did not reach the concept of rectilinear inertia and remained anchored to circular inertia because, unlike Descartes, he did not accept the consequences of a complete geometrization of the universe; this means that he did not accept the infinity of the universe and the destruction of the ordered cosmos.

# GALILEO

- While commenting on the numerous passages where Galileo seems to refer to rectilinear inertia, Koyré claims that the Italian scientist came close to conceive it, but refused to make the final step.
- Vergara Caffarelli has a clear purpose in his studies: 1) to prove, against Koyré, that the experimental aspect is fundamental in Galileo; 2) to demonstrate that Galileo clearly reached the concept of rectilinear inertia and refused that of circular inertia. (Vergara Caffarelli, "Il principio d'inerzia negli ultimi scritti di Galilei" 2007, p. 76). Boccaletti also claims that Galileo did not believe in circular inertia (Boccaletti 2016, *Galileo and the Equations of Motion,* chapter 4).



# GALILEO

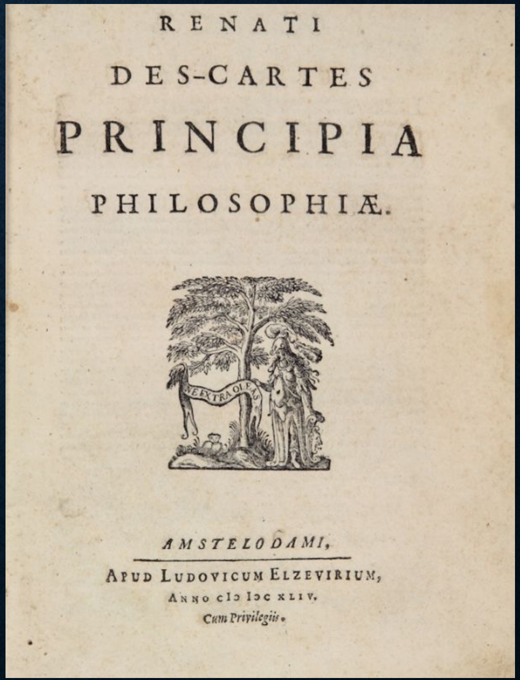
- Our interpretation: In the cosmological context – the motion of planets around the Sun and of planets around their axes – and in reference to a motion taking place on vast portions of the Earth’s surface, Galileo considered the uniform circular motion to be inertial. On the other hand, he was aware of the tendency to recede along the tangent in the local circular motion. Can these views be reconciled? Obviously, from the point of view of Newtonian physics they cannot. However, Galileo did not perceive the problem of reconciling two different views, because his ideas on circular motion were not clear. He did not study in depth this subject as, in contrast, he did for the free fall of heavy bodies or for the motion on inclined planes. Consequently, he did not achieve either a qualitative or a quantitative unitary view of the circular motion. Rather, his ideas on this subject were fragmentary.

# GALILEO

- He did not consider circular motion as a single unitary phenomenon. Thus, when observation or reasoning led him to think that the uniform circular motion should be conserved without the intervention of external forces, he adhered to this view. This was the case for the cosmic uniform circular motion and for the motion of a ship on a vast ocean or sea. For these motions Galileo never spoke of the tendency to recede along the tangent, though he did with regard to local uniform circular motion. The kind of coherence we demand of physics as a full-fledged science cannot be expected from physics in its nascent state. Therefore, according to Galileo uniform circular motion is inertial in some circumstances, while is non-inertial in other circumstances. However, this view had no influence when he developed concretely his mechanics in *Discorsi e dimostrazioni intorno a due nuove scienze*. In that case he dealt with local motions and, for such motions, his ideas were rather clear. As for the rectilinear inertia, we think that Galileo had quite a precise concept of it.

# DESCARTES

- In his *Principia Philosophiae* (1644, II, 37) Descartes stated a law which is very close to the principle of inertia.
- "The first law of nature: each and every thing, in so far as it can, always continues in the same state; and thus what is once in motion always continues to move. From God's immutability we can also know certain rules or laws of nature, which are the secondary and particular causes of the various motions we see in particular bodies. The first of these laws is that each thing, in so far as it is simple and undivided, always remains in the same state, as far as it can, and never changes except as a result of external causes".
- And:
- "The second law of nature: all motion is in itself rectilinear; and hence any body moving in a circle always tends to move away from the centre of the circle which it describes. The second law is that every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. This is true despite the fact that many particles are often forcibly deflected by the impact of other bodies; and, as I have said above, in any motion the result of all the matter moving simultaneously is a kind of circle".


RENATI
DES-CARTES
PRINCIPIA
PHILOSOPHIÆ.

AMSTELODAMI,
Apud Ludovicum Elzevirium,
Anno cIↃ IↃc xliv.
Cum Privilegiis.

# DESCARTES

- The situation is paradoxical: Descartes stated the inertia principle from a verbal standpoint. However, as a matter of fact, he did not use within his physics. Descartes was a pienist: he thought that the whole universe was full of matter. No rectilinear motion exists. He claimed that any motion is circular (curvilinear). Hence, the inertia law can never be actualized, either in principle. It is a tendency each particle holds, but which becomes never actual because no rectilinear motions exists. Given this conception and being Descartes' physics completely lacking of a mathematical treatment, it is difficult to claim what exactly the role of inertia was within his science. It plays a certain role in Descartes' theory of light diffusion, but it is not easy to guess exactly what it was. Descartes does not separate in a net manner the inertial components of motion form those which are not.

# DESCARTES

- Thus, although he claimed inertia laws, it is not evident its function within Descartes' theory and, surely, Descartes drew almost no consequence from inertia principle. This shows an important epistemological issue: it is not enough to claim a principle (in this case a physical principle) in abstract terms. For a proposition to become an axiom of physics, it is necessary to insert it within a deductive structure where all the consequences of such propositions are taken into account. This merit is a prerogative of Newton.
- There is a further doctrine of Descartes, which is necessary to take into account to understand Huygens' and Newton's conception of inertia.

# DESCARTES

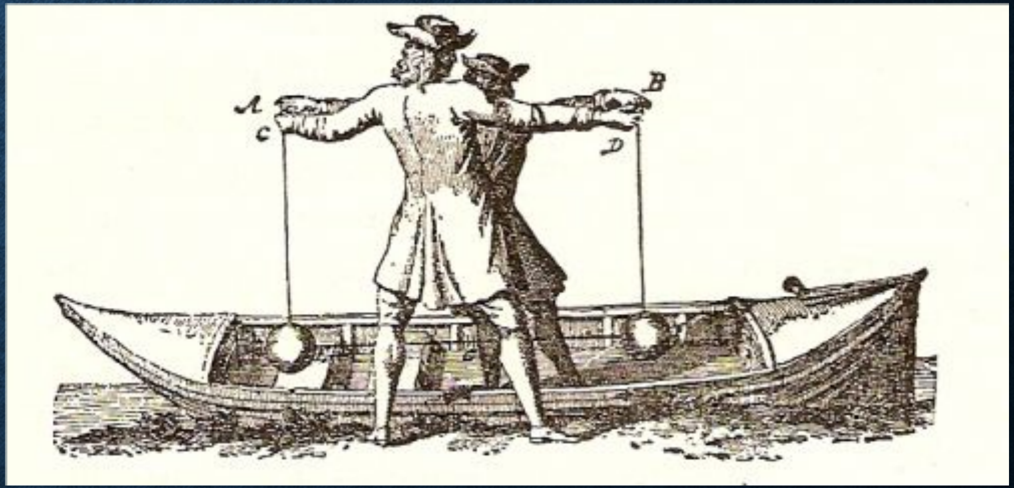
- Each motion is purely relative. The kinetic state of a body can be determined only in relation to the immediately neighbouring bodies.
- “In my definition I specified that the transfer occurs from the vicinity of contiguous bodies to the vicinity of other bodies; I did not say that there was a transfer from one place to another. This is because, as explained above, the term 'place' has various meanings, depending on how we think of it” (*Principia*, II, 28).
- “I further specified that the transfer occurs from the vicinity not of any contiguous bodies but from the vicinity of those which 'are regarded as being at rest'. For transfer is in itself a reciprocal process: we cannot understand that a body *AB* is transferred from the vicinity of a body *CD* without simultaneously understanding that *CD* is transferred from the vicinity of *AB*. Exactly the same force and action is needed on both sides. So if we wished to characterize motion strictly in terms of its own nature, without reference to anything else, then in the case of two contiguous bodies being transferred in opposite directions, and thus separated, we should say that there was just as much motion in the one body as in the other” (*Ibid*., II, 29).

# HUYGENS

- Christiaan Huygens' (1629-1695) ideas on motion evolved across three phases: in the first one, around 1652-1669, he thought every motion to be relative. Afterwards, from 1669 to 1687, he believed that circular motion and centrifugal forces might offer a criterion to distinguish absolute from relative motion. Finally, after the publication of Newton's *Principia*, Huygens rethought the whole problem of motion and came to the conclusion that every motion is relative. The third phase is the most interesting and original.

# HUYGENS

- The work *De Motu Corporum ex Percussione* (1656, posthumous 1703) is the first fundamental source to understand Huygens' ideas on motion. The presentation of this treatise is rather modern because Huygens poses three hypotheses that function as axioms and deduces from them his whole theory. The first hypothesis is the principle of inertia:
- "When a body is in motion, if nothing prevents its movement, it continues to move perpetually with the same velocity and along a straight line".
- Differently from Descartes, Huygens developed the collision theory in a mathematical manner. The principle of inertia is one of the axioms. Though this theory includes only a part of physics, this is an important improvement compared to the treatment of the previous scientists.

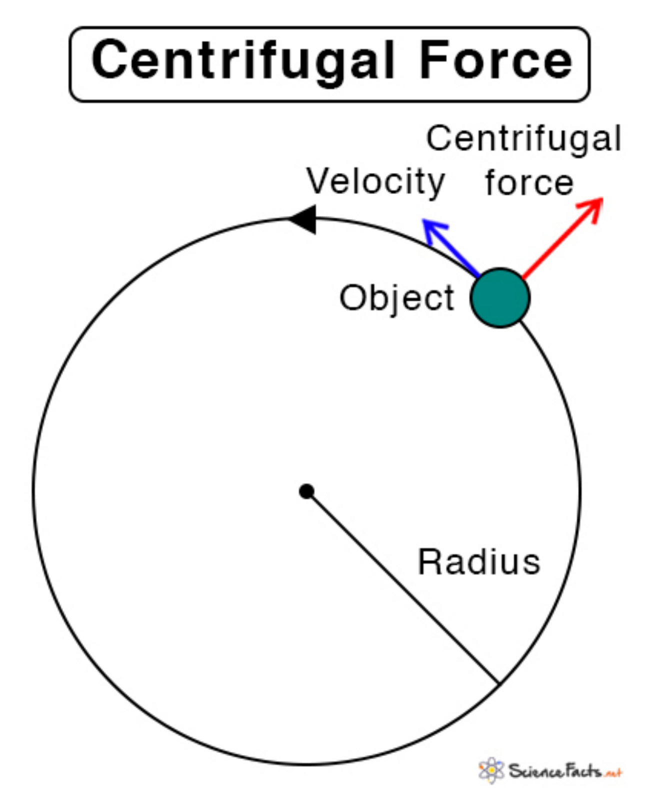


# HUYGENS

- In the published version of *De motu corporum ex percussione* the problem whether every motion is relative is not faced. Nonetheless, in a series of interesting notes (published in OH XVI as appendices to *De motu*) which date back to the period 1652-1654, Huygens states that all the motions are relative and that no absolute reference frame exists to which one could refer – at least in principle – the motion of bodies. Huygens claimed:
- "It is to be considered that what is moved, is moved with respect to other bodies with which it changes its distance and place. Analogously, what is at rest is at rest with respect to those bodies with which it maintains the same distance and position".
- This position is comprehensible. However, if every motion is relative, how is it possible to consider when a motion is uniform rectilinear and, hence, how is it possible to establish the inertia of a body?

# HUYGENS

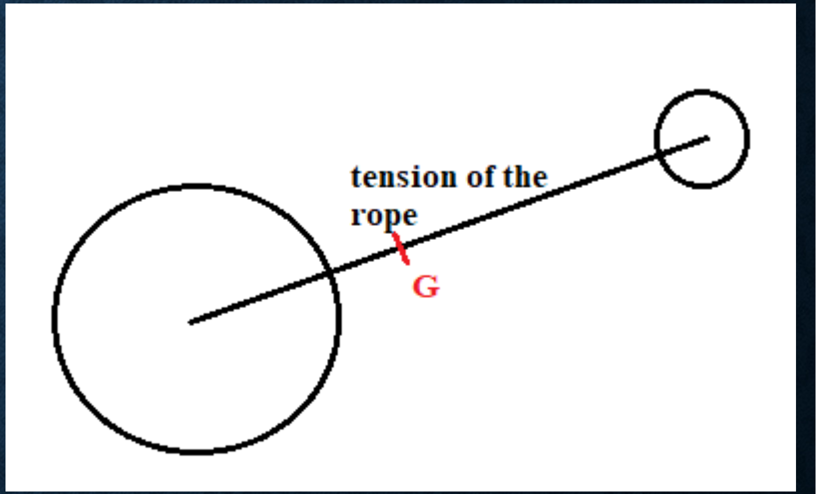
- Starting from around 1669, Huygens changed opinion with regard to the nature of motion and arrived at the idea that a criterion to distinguish absolute from relative motion can be found in circular movement. Such a criterion is the centrifugal tendency. The sources to reconstruct Huygens' thought on the problem of motion in the period 1669-1687 are scarce, but the conclusion that he identified the centrifugal force as a criterion of absolute motion is not conjectural. Indeed, in 1694 Leibniz wrote to Huygens: "It seems to me, however, that you yourself, Sir, in the past had the same mind as Mr. Newton about circular movement." Huygens confirmed Leibniz's assertion in a letter dated 24 August 1694: "As to absolute and relative motion, I admire your memory because you have remembered that in the past I had the same opinion as Mr. Newton with regard to circular motion".



# HUYGENS

- Huygens exemplifies the centrifugal forces-criterion by means of an ideal experiment similar to the one described by Newton at the end of the scholium to the *Definitions* in his *Principia*: given two globes connected by a rope, suppose them to be moved around their gravity centre *G*. The tension of the rope allows us to know the centrifugal force and, hence, the real movement of the globes.

- Why after the publication of Newton's *Principia* did Huygens change opinion? Our interpretation is that Newton had shown the absolute motion to be recognizable only in relation to an infinite, immobile mundane space. This connection reminded Huygens that he had claimed two theses: 1) it makes no sense to speak of a mundane space at rest; 2) centrifugal forces represent a criterion of absolute motion. Newton had proved that absolute motion has to be referred to a mundane space at rest. Huygens probably realized that his two theses are contradictory, and decided to maintain the first and to dismiss the second.



# HUYGENS

- Huygens did not deny the existence of an infinite mundane space, but he considered meaningless the idea to ascribe any kinetic state to it. Indeed, he judged the idea of rest and motion to be purely relative (i.e. it cannot be referred to absolute entities such as Newtonian space and time) and relational (concerning only the mutual kinetic state of more than one body).
- It is impossible to ascribe a kinetic state to space before having defined the notion of movement. But, as soon as we define motion as a relational concept, we discover that neither motion nor rest can be applied to space:
- "Motion cannot be understood but relatively. To what place? Maybe the immobile space? But what can we consider immobile, if we are still looking for the definition of movement?"

# HUYGENS

- There is, however, a serious candidate for absolute motion: it is circular motion, for dynamic rather than kinematic reasons. Indeed, to every circular motion a centrifugal force is associated that indicates inertia. In the second phase of his thought, Huygens himself regarded centrifugal forces as evidence of absolute motion. Let us see how he tackled the problem of circular (or more generally, curvilinear) motion in the third and last stage of his scientific career. Our thesis is that the weakness of his conception lies exactly in the inability to pass from the kinematic to the dynamic level.

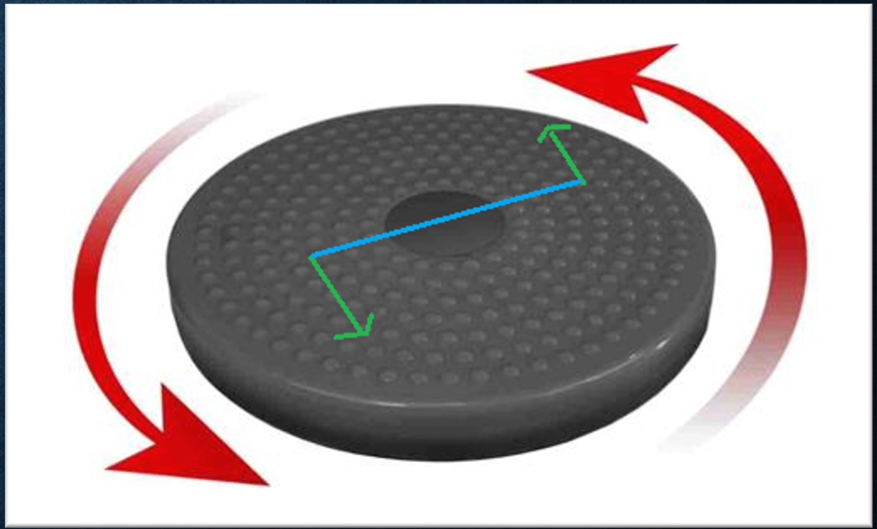
- The centrifugal force exists in any circular motion. It has various manifestations: if two bodies rotating around their gravity centre are linked by a rope, it is displayed by the tension of the rope; if a body is solid and rotates (for example, a disc), and you set something on top of it, this thing will fly away; if water rotates in a vessel, the water will rise on the vessel's walls.

# HUYGENS

- Huygens is not convinced that centrifugal forces are a sufficient criterion to determine absolute motion. He resorts to two arguments: firstly, he argues that even the circular motions of the parts of a rotating body are purely relative to the parts themselves. Furthermore, he states that a body, in addition to being animated by rotational motion, could also have a uniform rectilinear motion. But this kind of motion, by admission of the Newtonians themselves, can only be relative:
- “Given a single body or more joined bodies moving around a centre, it is possible to calculate through the centrifugal force how much circular velocity they received. However, this velocity is also relative to these bodies or to the parts of a single body. It is, instead, impossible to know how much true motion is in the single parts, namely, in reference to that space that they imagine to be  immobile. According to their own judgement, a rotating body, be it solid or composed of joined parts, can at the same time proceed rectilinearly with a movement common to all its parts. They admit that it cannot be recognized by any clue to what extent this rectilinear motion is true”.

# HUYGENS

- Huygens' new idea is that circular motion can be understood as the relative motion of the parts of a rotating body which are pushed in contrary directions Their motion cannot become rectilinear because a constraint or a link prevents it. A stone placed on a disc participates to the instantaneous motion of that part of the disc on which it is set, and, of course, the direction of its motion is along the tangent to the disc. Since the stone is not constrained, it flows away:
- "But this effect [the stone which flies away] only shows that, because of the impulse exerted on the circumference, the wheel's parts are pushed in different directions, with a relative motion of the ones with respect to the others. Thence, circular motion is only the relative motion of the parts which are pushed in opposite directions, when it is prevented by a constraint or a link".

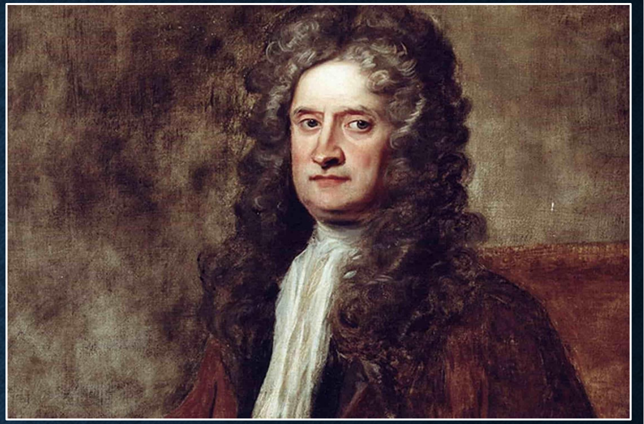


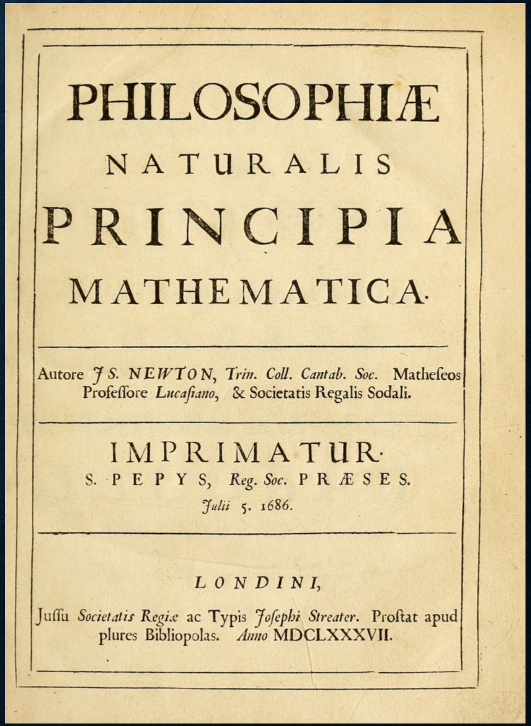
# HUYGENS

- Huygens’ arguments against absolute motion are correct from a kinematic point of view, but they miss the point in an important sense, since Newton introduced absolute space for a precise physical-dynamical reason. The reason is that, given a body, its inertia cannot be referred to neighbouring bodies, as the bucket experiment proves. Huygens regarded the velocities of the parts of a rotating body as reciprocal relations among these parts but the centrifugal force of a part, namely its inertia, cannot be referred to other parts as the bucket experiment – which Huygens knew – shows.
- In our book we develop many comparison with the secondary literature.

# NEWTON

- Isaac Newton (1642-1727) first faced explicitly the problem to determine the reference for inertia. He believed to have proved inertia to be referred to an absolute space. Conceptually his train of thought can be summarized through these steps:
- 1) Inertia principle.
- 2) Inertial reference frames.
- 3) Definition of absolute space and absolute motion. This does not mean that they exist. At the beginning they are only defined. Their existence is not given for granted.
- 4) Proof, by means of the celebrate bucket experiment, that Descartes was wrong: inertia cannot be referred to the neighbouring bodies and, in fact, to no body.
- 5) Thus it can only be referred to absolute space, which, hence, exists.





PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

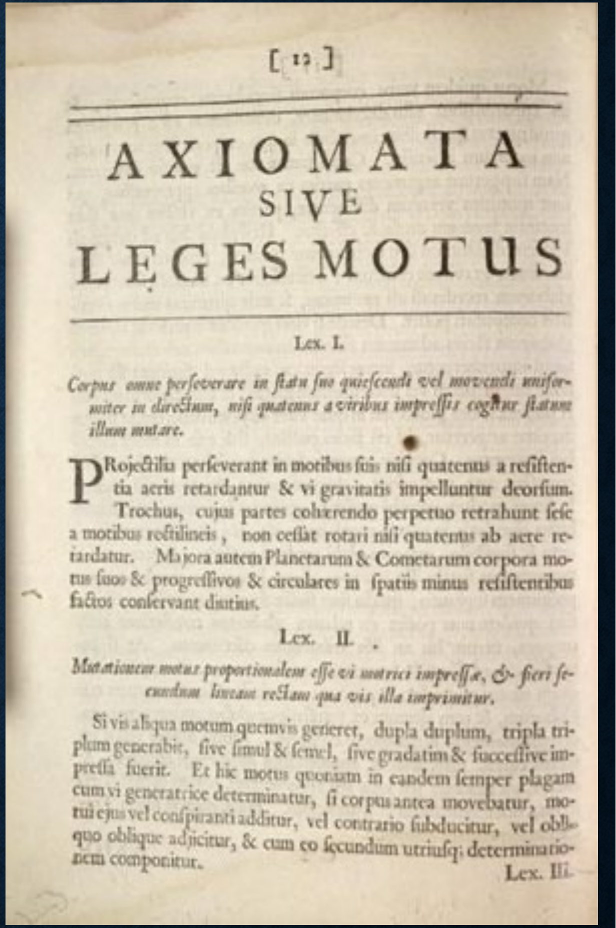
IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

# NEWTON

- Inertia principle. In the famous section of the first book of his *Principia*, entitled *Axioms or laws of movement* Newton states his three laws. The first one is the principle of inertia:

- 1) «Every body perseveres in its state of rest, or uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon».
- 2) «The alteration of motion is ever proportional to the motive force impressed; and it is made in the direction of the right line in which that force is impressed».
- 3) «To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts».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AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

*Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

Projectilia perseverant in motibus suis nisi quatenus a resistentia aeris retardantur & vi gravitatis impelluntur deorsum. Trochus, cujus partes cohaerendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatiis minus resistentibus factos conservant diutius.

Lex. II.

*Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimitur.*

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus quoniam in eandem semper plagam cum vi generatrice determinatur, si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel obliquo oblique adjicitur, & cum eo secundum utriusq; determinationem componitur.

Lex. III.

# NEWTON

- Concept of inertial reference frame. Newton clarifies that all the bodies at rest or moving rectilinearly and uniformly calculate the physical magnitudes in the same manner. They can be regarded the origin of inertial reference frames. In particular, they calculate the same acceleration and, hence, the same force (not the same velocities, as obvious). Since in the equations of motion only the second derivatives of space in respect to time enter, all the inertial reference frames see the world in the same manner. This is not the case for the not inertial ones.
- Corollary V to the Laws of Motion.
- “The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion”.

# NEWTON

- The problem faced by Newton is the following: since any motion we experience seems relative, is there the possibility to establish whether a body is at rest or is animated by a uniform rectilinear motion? This is very important because if the concept itself of inertia or inertial motion were relative,  the whole treatment of Newton’s physics would not be well founded. One might claim inertia to be a limit and operative concept: no motion is properly inertial because it is virtually impossible that all the forces acting on any body have a null resultant and momentum. However, in any physical circumstance, we know when a body can be considered at rest or moving of uniform motion. For example, while studying the movements on the Earth surface, one might claim the Earth to be an inertial reference frame because, from a practical point of view, no detectable error arises from this supposition. But in fact, the Earth is not inertial.
- This way of thinking is unsatisfactory for a natural philosopher as Newton (and as many physicists).

# NEWTON

- Definition of absolute time and space.
- "Absolute, True, and Mathematical Time, of it self, and from its own nature, flows equably without relation to anything external, and by another name is called Duration: Relative, Apparent, and Common Time, is some sensible and external (whether accurate or unequable) measure of Duration by the means of motion, which is commonly used instead of True time; such as an Hour, a Day, a Month, a Year".

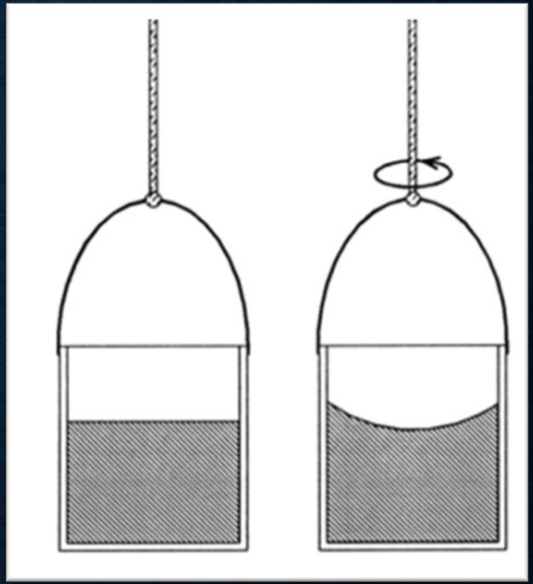
# NEWTON

«Absolute space, in its own nature, without regard to any thing external, remains always similar and immoveable. Relative space is some moveable dimension or measure of the absolute space; which our senses determine, by its position to bodies; and which is vulgarly taken for immoveable space. Such is the dimension of a subterraneous, an areal, or celestial space, determined by its position in respect to Earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the Earth, for instance, moves; a space of our air, which relatively and in respect of the Earth, remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be perpetually mutable […] Absolute motion, is the translation of a body from one absolute place into another; and Relative motion, the translation from one relative place into another».

Next steps: 1) to show that absolute motions exist; 2) that they are connected to inertia; 3) hence, that absolute space exists and has a physical reality.

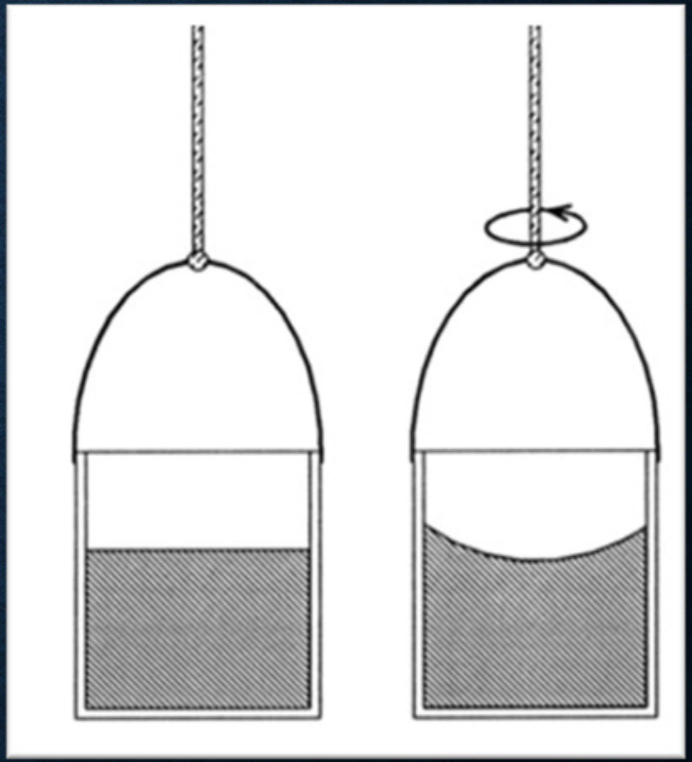
# NEWTON

- «The effects which distinguish absolute from relative motion are the forces of receding from the axe of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute motion, they are greater or less according to the quantity of motion. [in our interpretation this statement is like a theorem to be proved. What follows is the demonstration of the theorem. After the proof, Newton reaches the conclusion that absolute motion exists. Hence, absolute space exists] Be a vessel, hung by a long cord, so often turned about that the cord is strongly twisted Fill the vessel of water and hold it at rest together with the water. After by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move: but the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming itself into a concave figure (as I have experimented) and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavour to recede from the axe of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative and may be measured by this endeavour.



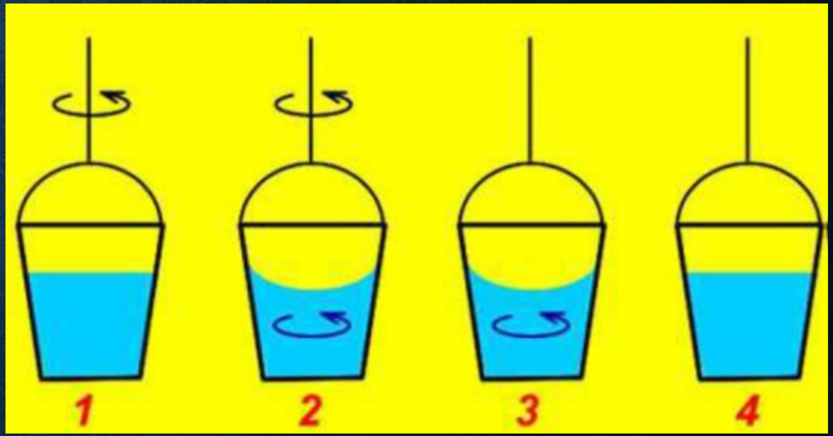
# NEWTON

- At first, when the relative motion of the water in the vessel was greatest, it produced no endeavour to recede from the axe: the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained of a plane surface [plane and at rest with respect to what? Newton answered: to the absolute space], and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel, proved its endeavour to recede from the axe; and this endeavour showed the real motion of the water perpetually increasing, till it had acquired its greatest quantity when the water rested relatively in the vessel. And therefore this endeavour does not depend upon any translation of the water in respect of the ambient bodies, nor can be true circular motion defined by such translations. There is only one real circular motion of any revolving body, corresponding to only one power of endeavouring to recede from its axe of motion».



# NEWTON

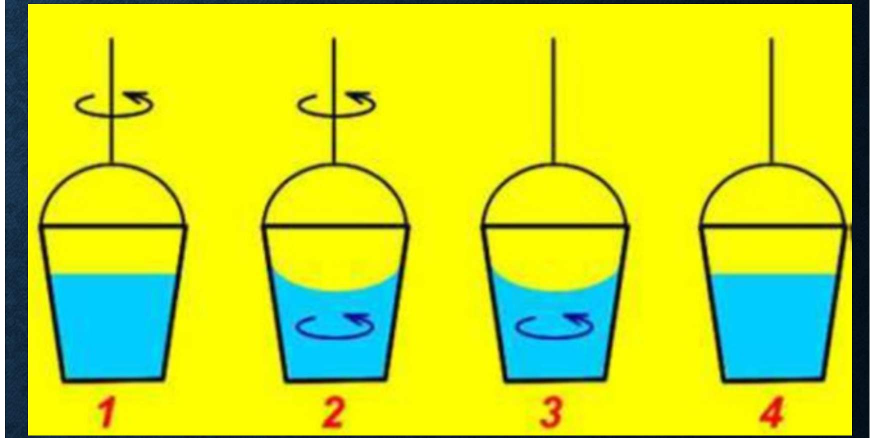
- Phases of experiment:
- 1) the rope of the vessel is rotated; the vessel is filled of water; the rope is unrolled; the vessel immediately moves, but the water at the beginning is at rest.
- 2) The water begins to rotate and to ascend on the bucket's walls (endeavour to recede from the centre). The circular motion of the water increases when its motion in respect to the vessel diminishes; because both the bucket and the water rotate.
- 3) The backet diminishes its rotation, while the water continues to rotate.
- 4) Finally, both the bucket and the water are at rest.



# NEWTON

| | Relative motion bucket/water | Water's surface | Centrifugal/inertial forces |
|---|---|---|---|
| 1. | Max | Flat | No |
| 2. | No | Curved | Yes |
| 3. | Max | Curved | Yes |
| 4. | No | Flat | No |

**There is no connection between the relative motions of the bucket and of the water. For, water's surface is flat both in the case the relative motion is Max (phase 1) and it is null (phase 4). Analogously, water's surface is curved both in the case of null relative motion (phase 2) and in case of maximal relative motion (phase 3).**



# NEWTON

- Rotation and ascent of the water=endeavour to recede from the axe=centrifugal force.
- The centrifugal force is an inertia force, an apparent force. The true force is the one which makes to unroll the rope of the vessel. For, if the water's particles were free to move, they would recede along the tangent with the velocity acquired at the beginning of the motion. This depends on the inertia principle (from here the name of inertial forces). However, they cannot exit from the vessel; hence they press vessel's walls and begin to ascend.
- What has proved Newton? Differently from what Descartes thought, the motion of the water, and hence, its inertia, cannot be referred to the neighbouring bodies and, in fact, to no body. This implies, for Newton, that the motion of the water is an absolute motion, hence referred to absolute space, to which, hence, inertia has to be referred.

# NEWTON

- Comparison with the literature.
- 1. Laymon (Newton's Bucket Experiment. *Journal of the History of Philosophy*, 16 (4): 399-413, 1978) claims
- A. Newton's statement according to which the centrifugal force cannot be determined with regard to the translation of the water with respect to the neighbouring (*ambientium*) bodies, refers only to the bodies which immediately surround the water, and not to distant bodies, such as the fixed stars. Authors such as Mach, Reichenbach and Nagel have, instead, interpreted Newton's claim as an assertion of the impossibility to refer the centrifugal forces to any body whatsoever, and this we believe to be the correct reading.
- B. "[My] interpretation of the bucket experiment, as showing an inverse instead of a direct relationship [between the motion of the bucket's walls and the water's], is somewhat different from the usual interpretation". His analysis is certainly new, but erroneous. There is no inverse or direct relation between the motion of the bucket's walls and that of the water, as can be clearly seen from our presentation of the bucket experiment.

# NEWTON

- 2. Rynasiewicz (By Their Properties, Causes and Effects: Newton's Scholium on Time, Space, Place and Motion. *Studies in History and Philosophy of Science*. First part 26, 1: 133-153. Second part 26, 2: 295-321 1995) claims that Newton gave for granted the existence of absolute motion and absolute space and that he, merely, aimed at proving that absolute motion is referred to absolute space.
- "Nowhere in the scholium does Newton seek to establish that each body has a state of true or absolute motion unique to it […].When Newton asserts that absolute motion and rest are distinguished from their relative counterparts by their properties, causes, and effects, he means to adduce reasons for believing absolute motion and rest are not special instances of the relative motion or rest of a body with respect to other bodies. The arguments that follow thus do not presuppose that absolute motion is by definition motion with respect to absolute space".

# NEWTON

- In a long introductory section ("Difficulties with the customary view"), the author highlights two problems concerning "the customary view": 1) if the aim of the bucket and of the two globes experiments were to prove empirically the existence of absolute motion, why did Newton redundantly propose two experiments aimed at the same purpose? This seems an implausible question. First of all, there is nothing strange in presenting more than one experiment to confirm a thesis. It is almost an ordinary procedure among scientists, even more justified when you are dealing with such an elusive phenomenon as absolute motion. Secondly, the two experiments have different nuances: the bucket experiment focuses on the effects which allow to distinguish the absolute from the relative motions, while the two globes experiment concerns the identification of an absolute force, displayed by the tension of the rope. 2) Rynasiewicz, then, points out that in all the parts of the scholium preceding the bucket experiment there is no evidence of the empirical existence of absolute motion. Again, this is true, but irrelevant. It depends on the fact that Newton first defined what the properties of absolute motion ought to be if such a motion existed and, afterwards, with the two experiments shows that it exists.

# NEWTON

- Were Newton's conclusions correct or were an overreading?
- 1) For sure, Newton was right that inertia cannot be referred to the neighbouring bodies.
- 2) He could conclude that inertia is to be referred to the set of all the inertial references frame. For, how is it possible to distinguish absolute space from other inertial frames? It would be necessary to come back to velocities and not accelerations. But this is Aristotelian physics, not Newtonian.
- 3) On the other hand, it is comprehensible that Newton assumed absolute space as a prototype of all the inertial reference frames.
- 4) Critics of Leibniz and Mach. Work of Lange at the end of the 19th century. Einstein.